MODELING VALUE AT RISK (VAR) POLICIES FOR TWO PARALLEL FLIGHTS OWNED BY THE SAME AIRLINE

Oki Anita Candra Dewi
Department of Engineering Management, Semen Indonesia School of Management (STIMSI), Gresik 61122 Indonesia / Department of Industrial Engineering, Institut Teknologi Sepuluh November (ITS), , E-mail: okianita@gmail.com

Ahmad Rusdiansyah
Department of Industrial Engineering, Institut Teknologi Sepuluh November (ITS), Surabaya 60111 Indonesia, E-mail: arusdian@ie.its.ac.id

Naning A. Wessiani
Department of Industrial Engineering, Institut Teknologi Sepuluh November (ITS), Surabaya 60111 Indonesia, E-mail: wessiani@ie.its.ac.id

ABSTRACT

This study develops a risk policy in Airline Revenue Management (ARM). ARM related to airline demand management policies to estimate and classify the various requests of pricing and capacity control. We made a model of the optimal policy value at risk (VaR) for two parallel flights owned by the same airline. The two parallel flights is a condition which an airline operates two flights in the same departure date with different time schedule. VaR is the worst possible losses under normal market conditions during the some period and certain confidence level. We specifically discuss the revenue risk policy for joint dynamic seat allocation problem including operational risks such as seat allocation risk. Moreover, in this proposed model, we also deal with passenger choice behaviour. We attempt to optimize VaR in determining seat allocation policies of both flight such as (i) improving revenue targets, (ii) choosing the best result for a given confidence level. To implement the model, we develop a dynamic programming algorithm for a set of expected revenues. We conduct some numerical experiments to show the behaviour of this model.

Keywords: Airline Revenue Management, Paralellel Flights, Seat Allocation Risk, Value at Risk

1 INTRODUCTION

Revenue management is also known as yield management. According to Talluri and Van Ryzin (2004) revenue management is related with demand management policies to estimate and classify the various request for pricing and capacity control as well as the entire system to make them. The primary goal of revenue management is to maximize revenue. However the problem in revenue management is when a request occur at this time, the request should be accepted now with the current price, or retained in anticipation of future price increases.

Revenue management widely adopted by a number of both manufacturing and service industries such as retail, automobile industry, cement industry, airlines and others. Application of revenue management in the airline known as Airline Revenue Management (ARM). ARM is often an important concern since its application used to anticipate demand uncertainty problems in the future due to excess inventory may not be stored and used in the next period, while a seats capacity offered always fixed and the fixed costs is high but marginal costs is low. Luo Li and Ji-Hua (2007) explains an airlines who implementing revenue management increase their revenue from 2% to 8%. American Airlines defines revenue management is maximize the revenue earned from selling seat inventory to the passenger in a timely manner. In addition, according to Dunleavy and Philiphs (2009) the purpose of revenue management is maximize total revenue obtained from total income of all passengers in accordance with the price of each class to the number of booking request in a particular class period.

Airline sells a ticket at different prices with the goal to maximize total revenue because the products services offered is a seat which has characteristics such as perishable products, namely products that have no residual value if it passes a certain period. That means, airlines will lost the opportunity revenue if the tickets were not sold until the flight depart. Different prices of tickets (fare class) will form a different sub-classes in the same flight and received the same services. Multi-fare classes are not a problem for customers as long as they are willing to pay for the tickets.

ARM is specifically discussed by several researchers. Subramanian et al., (1999) which discusses the amount of income over the period of reservations on a single flight condition. Lee and Hersh (1993) which split the period into discrete time where most widely one event will occur in each period. Luo Li and Ji-hua (2007) developed a model under competition using continuous time. Some papers discuss the situation where an airline may open more than one flight in the same departure date with different time schedule called parallel flights. Xiao et al., (2008) discuss the development of a model for parallel flight on dynamic pricing approach. Zhang and Cooper (2005) discuss the problems of dynamic seat allocation on parallel flight at the same airline. Chen et al., (2010) and Rusdiansyah et al. (2010) develop a model to optimize seat allocation on two parallel flight.

Various models from previous research about revenue management can be grouped in two categories: static models and dynamic models. The core of the static models is seek optimization of the number of seats offered at different prices sequentially ordered. Research on static models include Littlewood (1972) which set two different price classes in the same flight. In addition, Belobaba (1989) developed a study on multiple fare classes. While the dynamic model is a booking request in all fare classes througout the booking periode and dynamically decide whether the booking request is accepted or not. Dynamic models calculate the maximum expected revenue optimization for airlines. Research on the dynamic model have been carried out by researchers, some of which are Subramanian et al., (1999), Xiao et al., (2008), Chen et al., (2010). Revenue management that observe risks was first performed by Feng and Xiao (1999) who consider risks in term of sales due to price changes. Furthermore, Feng and Xiao (2008) integrate expectation utility theory into revenue management to support risk decisions. Koenig and Meissner (2011a) minimize the risk of failure with a set of target revenue values. Koenig and Meissner (2011b) calculate the optimal policy using standard deviation and value at risk (VaR). Jorion (2006) define VaR is a risk measurement that commonly used in finance to calculates the risk for a given probability level which is often referred to as confidence level.

However, none of previous paper discuss dynamic programming with the optimal policy value at risk (VaR) for two parallel flights owned by the same airline. In this study used revenue risk policy for joint dynamic seat allocation problem including operational risks such as seat allocation risk. Seat allocation control approach provide different seat proportion on each fare classes. Moreover, in this study also deal with passenger choice behavior and attempt to optimize VaR in determining seat allocation policies of both flights. Variables that changed for sensitivity analysis is the number of entities (the passengers), the proportion of flexible passenger, seat allocation for cheaper ticket prices, revenue targets and confidence level to produce the maximum expected revenue with the minimum risk.

2 MODEL DESCRIPTIONS

In this study, we discuss value-at-risk optimal policy model for revenue management problem on parallel flights that notice the passengers behavior in choosing flight schedules to maximize total revenue. The goal of this problem is minimize risk policy to obtain maximum total revenue. We develop a dynamic programming model to optimize value-at-risk policy by using a target value as practiced by Koenig and Meissner (2011a) and Koenig and Meissner (2011b) with adding a model of two parallel flight and passenger choice behavior as practiced by Chen et al. (2010). Two parallel flights according to Rusdiansyah et al. (2010) is two flights that serving a same route in different departure time. For example two parallel flights A and B, A has a scheduled flight departure earlier than B (A departure at 06.00 and B at 07.00). This study also calculates risk in certain given revenue targets.

Two parallel flight which opened on a different schedule with the same destination and offers a variety of classes with different ticket prices make the passengers have freedom to choose the flight schedule as their needs. This study uses a discrete time model approach as developed by Chen et al. (2010), Koenig and Meissner (2011a) and Koenig and Meissner(2011b). Optimal risk is use to obtain maximum total expected revenue of the two parallel flights.

2.1 Model Formulation

Seat capacity is denoted by C_k where k indicate the type of flight. C_1 is the seat capacity for flight 1 and C_2 seat capacity for flight 2. Remaining seats in time period n are given by c_1 and c_2. Each flight contained m fare class and expressed by i where $i = 1,2,..,m$.

r_i denoted as net income of class i on flight 1 and R_i is net income of class i on flight 2. Generally $r_1 > r_2 > r_3 > \cdots > r_m$ dan $R_1 > R_2 > R_3 > \cdots > R_m$. It shows fare class 1 is greater than the class 2, 3, until class m. The highest price class called high fare while the lower price is low fare. This research does not develop overbooking. Decision of booking request on passengers type j in fare class i would be processed to be accepted or rejected in each flight 1 and 2.

Table 1. Notation models

Notation	Description
m	Amount of fare class
ri	Net income in the first flight in fare class *i*
Ri	Net income in the second flight in fare class *i*
j	Type of passengers
p_{ji}^{n}	The probability of *j* request for the fare class *i* in period *n*
k	Flight number, $k = 1,2$
C_k	Seat capacity on the flight *k*
x_k	The number of booking request on the flight *k*
N	Booking period
X_N	Target revenue N period
$V_N^{\tilde{\pi}}((c1, c2, i), X_N)$	Minimum risk of failing target X_N for *c* remaining seats in fare class *i*
$W_n^{\tilde{\pi}^*}(c1, c2, x_n)$	Transformation value of $V_n^{\tilde{\pi}}$ with reducing fare class *i*
c_k	Remaining seats capacity on the flight *k*
α	Confidence level
$V@R_\alpha$	Value at risk in confidence level α
U	Revenue
F_{ki}	Fare class on the *k* flight in the fare class *i*
a	Indicator function $a = 0,1$

2.2 Determine Discrete Time

This research use a discrete time approach as developed by Subramanian et al. (1999). Discrete time will split booking request into smaller time period which only one event can occur. The possible events are: (1) customer arrival; (2) no any event occurred (null event) were represented by the probability of customer arrival in the fare class 0. The selling horizon for flight 1 and 2 is identical, written by [0,T]. Each smaller period declared by n, also called the stage. If N is the total number of decision periods, the period written backward from N to *0* which is the initial period and 0 the last period or departure, so the decision period declared by $n = N, N - 1, \ldots, 1, 0$.

2.3 Determine Event in Each Decision Period

The possible events may occur are:

(1) Customer arrival of type *j* with booking request on fare class *i*, *i=1,2,...,m;*

When the booking request occur, the airline will classify the type of passenger and decide to accept or reject the request on *i* fare class. The airline does not earn any income if the request rejected, otherwise if it accepted the airline earn revenue by *Fi > 0*. While type 3 of the passenger occur, the airline will determine which flight will be accommodate the request.

During the selling horizon, the three types of passengers can appear then the demand probability for fare class *i* with the passengers type *j* as follows:

$$\sum_{i=1}^{m}(p_{1i}^{n} + p_{2i}^{n} + p_{3i}^{n}) \leq 1$$

(2) No any event occurred (*null event*)

Null event represented by the probability of passenger on fare class *0*. The probability of no request for any type is given by $p_{1,0}^{n} = p_{2,0}^{n} = p_{3,0}^{n}$ and the sum of them will not more than 1. We use the following equation:

$$p_{1,0}^{n} + p_{2,0}^{n} + p_{3,0}^{n} \leq 1$$

With the both possible events, probability at each stage can be formulated as follows:

$$\sum_{i=0}^{m}(p_{1i}^{n} + p_{2i}^{n} + p_{3i}^{n}) \leq 1 \ldots\ldots\ldots..(1)$$

2.4 Determine Probability of Failing Target Revenue

Applying the model developed by Koenig and Meissner (2011a) and based on the concept of Chen et al. (2010) to our model, we get the probability of failing target revenue two parallel flight as follows:

$$V_0^{\pi^*}((c1, c2, i), x_0) = \begin{cases} 1 & x_0 > 0 \\ 0 & x_0 \leq 0 \end{cases}$$

$$V_n^{\pi^*}((c1, c2, i), x_n) =$$

$$\min_{a\in(0,1)} \{ \left(\sum_{j=0}^{k} p_{1,j}^{n-1} V_{n-1}^{\pi^*}((c1 - a, c2, j), x_n - aF_{1i}) \right)$$

$$+\left(\sum_{j=0}^{k} p_{2,j}^{n-1}\, V_{n-1}^{\pi^*}\big((c1, c2-a, j), x_n - aF_{2i}\big)\right)$$

$$+\left(\sum_{j=0}^{k} p_{3,j}^{n-1} \min\left\{V_{n-1}^{\pi^*}\big((c1-a, c2, j), x_n - aF_{1i}\big); V_{n-1}^{\pi^*}\big((c1, c2-a, j), x_n - aF_{2i}\big)\right\}\right)\Big\} \ldots\ldots (2)$$

Refers to Koenig and Meissner (2011a), the formulation of dynamic programming in Equation 2 is require transformation to reduce the scope of the state . Transformation is denoted by $T_n(c1, c2, x) \coloneqq \sum_{i=0}^{k}\big(p_{1,i}^n + p_{2,i}^n + p_{3,i}^n\big)\, V_n(c1, c2, i, x)$. The value function $W_n(c1, c2, x) \coloneqq T_n(c1, c2, x) V_n(c1, c2, i, x)$ stand for reduce the scope of fare class *i*. So the equation 2 is transformed as:

$$W_0^{\pi^*}\big((c1, c2), x_0\big) = \begin{cases} 1 & x_0 > 0 \\ 0 & x_0 \leq 0 \end{cases}$$

$$W_n^{\tilde{\pi}^*}(c1, c2, x_n) = T_n(c1, c2, x_n) V_n^{\tilde{\pi}^*}(c1, c2, i, x_n)$$

$$= \sum_{i=0}^{k}\big(p_{1,i}^n + p_{2,i}^n + p_{3,i}^n\big) \min_{a \in A}\left\{W_{n-1}^{\tilde{\pi}^*}\big(c1-a, c2-a, x_n - aF_{ji}\big)\right\} \ldots\ldots (3)$$

2.5 Develop value-at-risk (VaR) for a Given Confidence Level

Based on a model developed by Koenig and Meissner (2011b) by adding two parallel flight condition and pay attention to the passengers behavior, the new equation is:

$$V@R_\alpha^\pi\left(\sum_{j=0}^{n} r_j\right) = \inf\left\{u: \sum_{i=0}^{k}\big(p_{1,i}^n + p_{2,i}^n + p_{3,i}^n\big)\left(\sum_{j=0}^{n} r_j \leq u\right) \geq \alpha\right\} = \inf\{u: W_n^\pi(c1, c2, u) \geq \alpha\} \ldots (4)$$

3 NUMERICAL EXPERIMENT

We evaluated the proposed computation method by the similar model introduced by Koenig and Meissner (2011a). In the model performed several numerical experiments and calculate every stage and state. The optimal output at *n-1* will be an input on the next stage *n*. In case 1, we examine models to changes value of target revenue with the parameters of each flight are identical. Case 2, we also examine models to changes value of target revenue but the parameters of each flight are different. And the third case, we examine models againts the risk of failure to achieve expected revenue from equations developed by Chen et al., (2010).

Case 1

In this case, we use initial parameters which can be seen on table 2 and 3. Total period used in this case is N = 30. Fare class and the probability for both flights are equal. Value of target revenue is altered at \$1200, \$1220, \$1400 and \$1500.

Table 2. Fare class *i* and the probability of request from *n=1* to *n=30* in case 1

class i	F1(i)	F2(i)	n=0			1<=n<=4			5<=n<=11			12<=n<=18			19<=n<=25			26<=n<=30		
			p1	p2	p3	p1	p2	p3	p1	p2	p3	p1	p2	p3	p1	p2	p3	p1	p2	p3
0	0	0	0,33	0,33	0,33	0,23	0,23	0,23	0,13	0,13	0,13	0,20	0,20	0,20	0,20	0,20	0,20	0,19	0,19	0,19
1	200	200	-	-	-	0,05	0,05	0,05	0,05	0,05	0,05	0,03	0,03	0,03	0,02	0,02	0,02	0,03	0,03	0,03
2	150	150	-	-	-	0,05	0,05	0,05	0,05	0,05	0,05	0,03	0,03	0,03	0,02	0,02	0,02	0,03	0,03	0,03
3	120	120	-	-	-	-	-	-	0,05	0,05	0,05	0,03	0,03	0,03	0,05	0,05	0,05	0,05	0,05	0,05
4	80	80	-	-	-	-	-	-	0,05	0,05	0,05	0,03	0,03	0,03	0,05	0,05	0,05	0,05	0,05	0,05

Source: Koenig and Meissner (2011a)

Table 3. Parameters

Parameters	Value
N	30
m	4
Capacity 1	5
Capacity 2	5
XN	1200

The probability at the last period n = 0 are zero for all classes on flight F1 (i) and F2 (i), where *i = 1,2, ... 4*. However, when the artificial class 0, the probability are one. The results of case 1 as follows:

Table 4. Result of $W_n^{\tilde{\pi}^*}$on case 1

Target value *XN*	1200	1220	1400	1500
$\boldsymbol{W_n^{\tilde{\pi}^*}}$	0.088	0.101	0.336	0.529

In the table above, it can be seen that the higher the revenue target, the greater the probability of failure to reach the target value. The values $V@R_\alpha$ dan $W_n^{\tilde{\pi}^*}$ represent a probability. The results of this case, there is no $V@R_{10\%}$ but the approach is $V@R_{10,1\%}$ on target value \$1220. By using 1000 times simulation, $\boldsymbol{V@R_{10\%}}$ with target revenue \$1200 can be seen in the following graph:

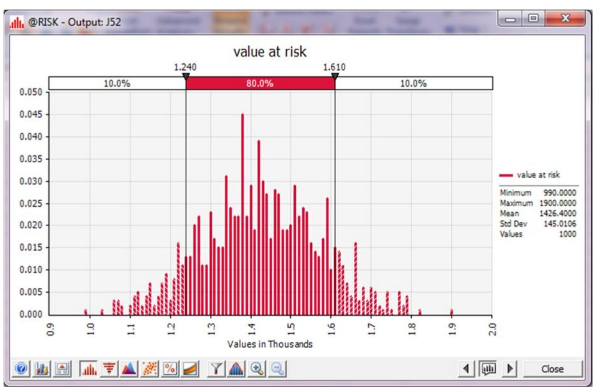

Figure 1. Graph $V@R_{10\%}$ in case 1

The histogram shows minimum value of revenue that occur is equal \$990 and maximum of \$1900. At the confidence level α = 10% obtained value of \$ 1,240. Every generate simulations, the value can be different from each other with a standard deviation of 145.

Case 2

In this case, we use initial parameters which can be seen on table 5. Total period used in this case is N = 30. Fare class and the probability for both flights are equal. Value of target revenue is altered at \$1200, \$1220, \$1300, \$1400 and \$1500.

Table 5. Fare class *i* and the probability of request from *n=1* to *n=30* in case 2

class i	F1(i)	F2(i)	n=0			1<=n<=4			5<=n<=11			12<=n<=18			19<=n<=25			26<=n<=30		
			p1	p2	p3	p1	p2	p3	p1	p2	p3	p1	p2	p3	p1	p2	p3	p1	p2	p3
0	0	0	0,33	0,33	0,33	0,23	0,23	0,23	0,13	0,13	0,13	0,16	0,16	0,16	0,20	0,20	0,20	0,13	0,13	0,13
1	200	200	-	-	-	0,03	0,05	0,02	0,03	0,03	0,05	0,06	0,04	0,03	0,02	0,02	0,02	0,01	0,02	0,05
2	150	150	-	-	-	0,04	0,10	0,06	0,04	0,08	0,04	0,07	0,04	0,01	0,02	0,02	0,02	0,01	0,06	0,06
3	120	120	-	-	-	-	-	-	0,05	0,05	0,07	0,07	0,04	0,03	0,05	0,05	0,05	0,02	0,07	0,08
4	80	80	-	-	-	-	-	-	0,06	0,07	0,03	0,07	0,04	0,03	0,05	0,05	0,05	0,05	0,09	0,10

The results of case 1 as follows:

Table 6.Result of $W_n^{\tilde{\pi}^*}$ on case 2

Target value *XN*	1200	1220	1300	1400	1500
$W_n^{\tilde{\pi}^*}$	0.090	0.108	0.210	0.404	0.654

Case 2 also using 1000 times simulation and target revenue was set to $1400, $V@R_{10\%}$ with target revenue can be seen in the following graph:

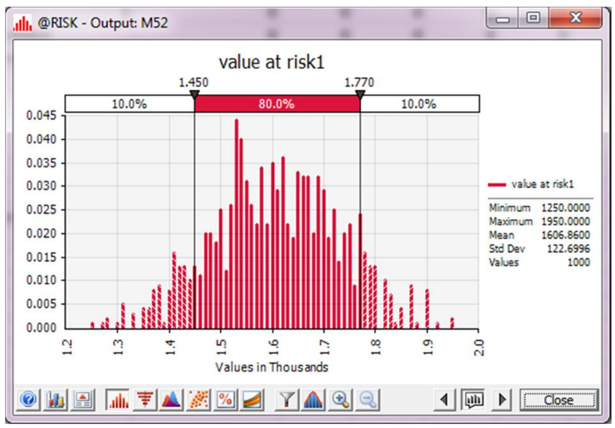

Figure 2. Graph $V@R_{10\%}$ in case 2

The histogram on picture 2 shows minimum value of revenue that occur is equal $1,250 and maximum of $1,950. At the confidence level α = 10% obtained value of $ 1,450. Every generate simulations, the value can be different from each other with a standard deviation of 122.7.

Case 3

In case 3 the parameters of this model use N = 30 and seat capacity was set by 5 at each flight. Ticket price set only two fare classes, class 1 for the most expensive and class 2 for cheapest. In addition, ticket prices for flight 1 set more expensive than flight 2. By using equations developed by Chen et al., (2010) and Rusdiansyah et al. (2010), we get the expected revenue $1542. Further, this output will be used as target revenue to compute how much risk of failure from the expected revenue. The computation result in table 7.

Table 7. Risk combination with target revenue $1542

	C2					
C1	0	1	2	3	4	5
0	1	1	1	1	1	1
1	1	1	1	1	1	1
2	1	1	1	1	1	1
3	1	1	1	1	1	0.994
4	1	1	1	1	0.993	0.745
5	1	1	1	0.992	0.707	0.302

Source: VBA Excel Computation

Table 7 show combination of seat allocation againts risk failure to achieve the target. The total expected revenue have risk of failure by 30,2%. further experiment conducted by changing seat capacity of each flight is 6, so all seat on offer has a total of 12 seats. The optimal expected revenue reach to $1817. With the same way, risk combination of failing the target can be seen as the following table:

Table 8. Risk combination with target revenue $1817

	C2					
C1	0	1	2	3	4	5
0	1	1	1	1	1	1
1	1	1	1	1	1	1
2	1	1	1	1	1	1
3	1	1	1	1	1	0.958
4	1	1	1	1	0.95	0.723
5	1	1	1	0.94	0.687	0.304

Source: VBA Excel Computation

The risk when total seat offered $Ck = 10$ equal to 30,2% have nearly same with $Ck = 12$ equal to 30,4%. This condition shows when the target revenue is an optimal expected revenue, the risk of failure is not differ significantly. Figure 3 shows the comparison of this experiment.

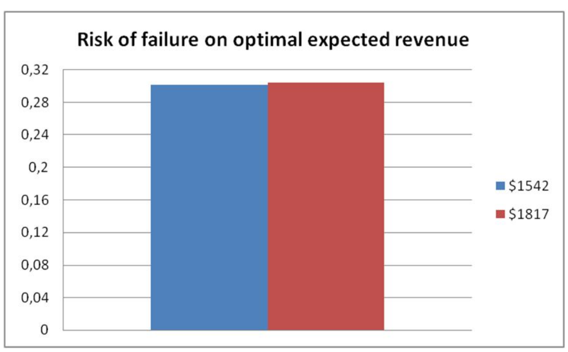

Figure 3. Comparison Risk of Failure

4 CONCLUSIONS

This research has developed a computational approach to calculate the optimal value at risk policy for the revenue management problem on two paralell flights owned by the same airlines. This study use a dynamic capacity control model that is part of the quantity-based revenue management for joint dynamic seat allocation problem including seat allocation risk and also deal with passenger choice behavior that attempt to optimize VaR in determining seat allocation policies of both flights.

Given a confidence level α, the proposed method compute possible value at risk results and select the best result in accordance with confidence level. This study also proposes an approach to reduce the calculation to obtain the optimal value at risk.

Result of numerical experiment, we obtained the same risks for a single flight and two paralell flight when the total seat capacity and parameters offered is the same. Computation of risk did not differ significantly when applied the optimal of total expected revenue as revenue targets. The calculation of the optimal policy value at risk can be used for other revenue management models such as dynamic pricing if revenue targets are known.

5 REFERENCES

Belobaba, P. P. 1989. Application of a probabilistic decision model to airline seat inventory control. Operations Research, 37, 183–197.

Chen, S., Gallego, G., Li, M. Z. F. & Lin, B. 2010. Optimal seat allocation for two-flight problems with a flexible demand segment. European Journal of Operational Research, 201**,** 897-908.

Curry, R. E. 1990. Optimal airline seat allocation with fare classes nested by origins and destinations. transportation Science, 24,193-204.

Dan Zhang & Cooper, W. L. 2005. Revenue management for paralel flight with customer choice behavior. Operation reasearch, vol. 53 no.3 pp. 415-431.

Dunleavy H & Philiphs, G. 2009. The future of airline revenue management. Journal or revenue management.

Feng, Y. & Xiao, B. 1999. Maximizing revenues of perishable assets with a risk factor. Operations Research, 47(2) 337–341.

Feng, Y. & Xiao, B. 2008. A risk-sensitive model for managing perishable products. Operations Research, 56(5) 1305–1311.

Harper, D. Introduction to Value at Risk (VaR), Investopedia, 2004, URL : www.investopedia.com.

Jorion, P. 2006. Value at Risk: The New Benchmark for Managing Financial Risk, McGraw-Hill.

Koenig, M. & Meissner, J. 2011a. Risk Minimizing Strategies for Revenue Management Problems with Target Values. Lancaster University Management School.

Koenig, M. & Meissner, J. 2011b. Value-At-Risk Optimal Policies for Revenue Management Problems. Lancaster University Management School.

Lancaster, J. 2003. The financial risk of airline revenue management. Journal of Revenue and Pricing Management, 2(2) 158–165.

Lee, T. C. & Hersh, M. 1993. A Model for dynamic airline seat inventory control with multiple seat bookings. transportation science, 27(3) 252-265.

Littlewood, K. 1972. Forecasting and control of passenger bookings. AGIFORS Symposium Proceedings, 12, 103-105.

Luo Li & Ji-Hua, P. 2007. Dynamic Pricing Model for Airline Revenue Management Under Competition. Systems Engineering - Theory & Practice, Vol 27, 11.

Rothstein, M. 1971. An Airline Overbooking Model. Transportation Science, Vol. 5, No. 2, pp. 180-192.

Rusdiansyah, A., Wessiani, N. A., Pradana, H. & Mariana, D. 2010. Joint Dynamic Pricing Model for Two Parallel Flights Considering Overbooking, Cancellations, and No-Show Customers. Eastern Asia Society for Transportation Studies, Vol.8, 2011.

Subramanian, J., Stidham, J. R. & Lauthenbacker, C. J. 1999. Airline yield management with overbooking, cancellation, and no show. transportation science, vol. 33 no. 2.

Suzuki, Y. 2006. Net Benefit of Airline Overbooking. Transportation Research, Part E no. 42 . 1-19.

Talluri, K., T., & Van Ryzin G. 2004. The theory and practice of revenue management., Boston, kluwer academic publisher.

Weatherford, L. R. 2004. EMSR versus EMSU: Revenue or utility? Journal of Revenue and Pricing Management, 3(3) 277–284.

Xiao, Y.-B., Chen Jian & Ling, X. 2008. Joint Dynamic Pricing for two parallel flights based on passenger choice behavior. System Engineering Theory & practice, vol. 28 pp. 46-55.

Zhang, D. & Cooper, W. L. 2005. Revenue management for paralel flight with customer choice behavior. Operation reasearch, vol. 53 no.3 pp. 415-431.