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The Sustainable Economic Order Quantity Model: A Model Consider Transportation, Warehouse, Emission Carbon Costs, and Capacity Limits

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Abstract— Recently, the problem of minimizing emission carbon in the industry has become a focus of much research. One of the efforts that were successfully carried out was managing the sustainable economics order quantity (SEOQ). However, several SEOQ had assumption such as (1) Transportation and warehouse costs are ignored, (2) Transportation emissions are avoided, and (3) warehouse Capacity Limit are ignored. In this paper, we developed the SEOQ model with warehouse costs and capacity limits. There are two proposed SEOQ models. Model 1 was the SEOQ model with Transportation and warehouse costs also emission carbon. At Model SEOQ 2, this model considered Transportation and warehouse costs, emission carbon, and capacity. We also proposed procedures to minimize total inventory costs. Finally, a numerical experiment was conducted to test the model. Based on numerical experiments, the proposed model useful to solve SEOQ problems with Transportation and warehouse also costs capacity limits.
Keywords; Capacity; SEOQ; Transportation; Warehouse

1. Introduction

Inventory has an essential role in the supply chain [1]. In addition, it is the primary key for the company to support the smooth production [2, 3]. It has an impact on costs [4]. Several approaches have been proposed by experts to complete the inventory problem. These include the Economic Order Quantity (EOQ) [5], dynamic programming [6, 7], and the heuristic algorithm [8]. The EOQ model developed by Harris [5] is referred to as a primary inventory model that inspires researchers. It is more attractive to researchers than other models. Other methods require complicated efforts to solve problems. In the basic EOQ model, several researchers have developed this model by considering transportation costs. These focus on economic aspects.

Many efforts have been made by researchers to develop the EOQ model. Some researchers consider transportation costs in EOQ. Baumol and Vinod [9] were one of the researchers who developed the

EOQ model by considering transportation costs. Their model aims to minimize total transportation, ordering, and transportation costs. The Harris [5] model was then developed by add stock out costs by Buffa and Reynolds [10]. Recently, the problem of inventory in consideration of environmental problems has become a concern of many researchers. It is referred to by researchers as a problem of sustainable inventory [11].

Some experts consider the environment as a problem in companies and the world. Through government regulations, companies are required attention in environmental aspects. The environmental aspect includes emissions and waste. In developed countries, the government provides a carbon emission tax policy to companies. It is given so that companies care to environmental issues [12-14]. Carbon emissions in inventory problems arise from several activities. It includes transportation, storage, and warehouse. Through inventory management, companies can control carbon emissions and economic aspects [15-17]. Experts have been investigated the sustainable EOQ (SEOQ) issue. Generally, environmental performance considered is carbon emissions. Some researchers have investigated this problem including Chen, et al. [18], Jaber, et al. [19], and Taleizadeh, et al. [20]. Generally, they consider the issue of carbon emissions consequences of the order frequency aspect and the quantity of storage. In addition, several studies on SEOQ issues with consideration of transportation costs and carbon emissions have been carried out by researchers. Battini, et al. [21] developed the SEOQ model by considering internal da external transportation.

Several studies on the issue of SEOQ have been carried out by researchers. However, the studies still used several assumptions. These include unlimited warehouse capacity and no warehouse costs. Warehouse costs are needed because the company does not have a warehouse. Companies use it for storing goods. This study aims to develop the SEOQ model with warehouse costs and capacity limits. There are two proposed SEOQ models. Model 1 was the SEOQ model with the cost of transportation and warehouse as well as carbon emissions. In the SEOQ 2 model, this model considers the costs of transportation and warehouse, carbon emissions, and capacity. We also propose procedures to minimize total inventory costs.

2. Methods

2.1 Assumption

The proposed model has several assumptions. These include (1) the demand for each period is constant and deterministic, hence, the demand annual is fixed, (2) transportation costs, holding costs, social costs of vehicle emissions, fixed waste costs per waste disposal activity and warehouse costs are fixed, (3) distance traveled, average speed, proportion of waste is fixed. (4) warehouse capacity does not change, (5) costs owned are unlimited, and (6) models are used to complete single products. In this paper, the proposed SEOQ model was carbon emissions. In addition, we developed the EOQ model by considering warehouse capacity constraints. The EOQ model was developed with carbon emissions based on the model proposed by Battini, et al. [21]. The proposed SEOQ model considering carbon emissions with the effect of carbon emissions; waste produced and transportation. The notations used in this model are as follows:

A : Order Cost

H : Inventory Cost

a : fixed costs for each trip

b : Variable Costs per Unit

d : mileage

α : proportion of return the demand

D : Demand

λ : Lagrange

β : social costs of vehicle emissions

v : speed average

γ : Waste costs (Rp/unit)

g : Warehouse Costs

F : Maximum storage capacity

Q : economical order quantity

Q_0 : classical economic order quantity

$Q_{(F)}$: economical order quantity with warehouse restrictions

$C(Q)$: Purchase Costs

$C_r(Q)$: Transportation Costs per cycle

$C_e(Q)$: Emission Costs per cycle

$C_w(Q)$: Waste produced per cycle

γ_0*:* fixed cost per waste disposal activity

θ*:* The proportion of waste produced per lot Q

o*:* Use of space per unit

$C_h(Q)$*:* Warehouse Cost

TC *:* Total inventory costs

$TC_{(h)}$*:* Total inventory costs with warehouse costs

2.2 Basic Sustainable EOQ Model

Some of the cost components of the basic SEOQ model include purchasing costs, transportation costs, emissions costs, and waste costs. Formula purchase per cycle can be seen in equation 1. Transportation costs per cycle (shipping and collection of returned goods) are described in equation (2). Emission costs for transportation per cycle are seen in equation (3). Waste produced by the inventory system per cycle is illustrated in equation (4). The mathematical model for calculating the total cost of inventory per cycle was to add the entire cost component in equation (1), equation (2), equation (3), and equation (4). The total inventory cost per cycle can be seen in equation (5), $T = Q/D$ Show cycle time. Therefore, the annual total inventory cost (TC) is seen in equation (6). To obtain the optimal Q value, we make the first equation (6) towards Q. Therefore, we get the optimal Q solution as in equation (7).

$$C(Q) = A + CQ + h\frac{Q^2}{2D} \tag{1}$$

$$C_r(Q) = 2a + bdQ + bd\alpha D\frac{Q}{D} = bdQ + bd\alpha Q \tag{2}$$

$$C_e(Q) = 2\beta\frac{d}{v} \tag{3}$$

$$C_w(Q) = \gamma_0 + \gamma Q(\theta + \alpha) \tag{4}$$

$$TC = \frac{[C(Q) + C_r(Q) + C_e(Q) + C_w(Q)]}{T} \tag{5}$$

$$TC = \frac{A\,D}{Q} + c\,D + \frac{h\,Q}{2} + \frac{2\,a\,D}{Q} + b\,d\,D + 1 + \alpha + \frac{2\beta\,d\,D}{v\,Q} + \frac{\gamma_0\,D}{Q} + \gamma(\theta + \alpha)\,D \tag{6}$$

$$Q = \frac{\sqrt{2}\sqrt{D}\sqrt{(A + \gamma_0 + 2a)v + 2\beta d}}{\sqrt{h}\sqrt{v}} \tag{7}$$

2.3 The proposed sustainable EOQ model 1

Based on the basic SEOQ model, we developed the proposed SEOQ model 1 with add warehouse costs. It is a warehouse cost to minimize total inventory costs. The warehouse cost formula is illustrated in equation (8). Furthermore, the annual total inventory cost formula with warehouse costs ($TC_{(h)}$) was to add the entire cost component in equation (1), equation (2), equation (3), and equation (4). The total inventory cost per cycle can be seen in equation (5), and equation (8). Therefore, the formula was formulated in equation 9. Equation (9) was first derived from the value of Q. Through the first derivative of equation (9) to the value of Q, we get the optimal formula Q as in equation (10)

$$C_h(Q) = Qgo \tag{8}$$

$$TC = \frac{A\,D}{Q} + c\,D + \frac{h\,Q}{2} + \frac{2\,a\,D}{Q} + b\,d\,D + 1 + \alpha + \frac{2\beta\,d\,D}{v\,Q} + \frac{\gamma_0\,D}{Q} + \gamma(\theta + \alpha)\,D + Qgo \tag{9}$$

$$Q = \frac{\sqrt{2}\sqrt{D}\sqrt{(A + 2a + \gamma_0)v + 2\beta d}}{\sqrt{v}\sqrt{2go + h}} \tag{10}$$

2.4 Proposed Sustainable EOQ Model 2 (Warehouse Capacity Limitation)

In developing the proposed model with this warehouse capacity limitation, equation (9) is added to the function of the warehouse constraint constraints. We used the Lagrange method to solve warehouse constraints. At the warehouse boundary, materials have dimensions denoted by o. the number of economic orders (Q) is not allowed to exceed warehouse capacity. Warehouse boundary formula was seen in equation (11). The Lagrange formula was generated by summing equation (9) and equation (11). This formula is illustrated in equation (12). The Solution to the Lagrange formula (equation (12)) was done by performing the first partial derivative of λ and Q. The result of the partial derivative of equation (12) toward λ is seen in equation (13). Furthermore, the results of the partial derivative of equation (12) toward Q is illustrated in equation (14). To obtain the value of the Lagrange variable (λ), substitute equation (14) into equation (15). The result is illustrated in equation (15).

$$Q.o \leq F \tag{11}$$

$$TC = \frac{A\,D}{Q} + c\,D + \frac{h\,Q}{2} + \frac{2\,a\,D}{Q} + b\,d\,D + 1 + \alpha + \frac{2\,\beta\,d\,D}{v\,Q} + \frac{\gamma_0\,D}{Q} + \gamma\,(\theta + \alpha)\,D \\ + Q\,g\,o + \lambda(Qo - F) \tag{12}$$

$$Q = \frac{F}{o} \tag{13}$$

$$Q_{(F)} = \frac{\sqrt{2}\,\sqrt{D}\,\sqrt{(A + \gamma_0 + 2a)\,v + 2\beta d}}{\sqrt{v}\,\sqrt{(2g + 2\lambda)\,o + h}} \tag{14}$$

$$\lambda = \frac{2\big((A + \gamma_0 + 2a)\,v + 2\beta d\big)\,Do^2 - 2gF^2vo - hF^2v}{2F^2vo} \tag{15}$$

2.5 Procedures to minimize total inventory costs with warehouse capacity limits

We proposed a procedure to describe the most suitable model to solve the problem. The procedure was based on the SEOQ 1 model and the proposed SEOQ model 2 with capacity constraints. The procedure is as follows: 1). Calculate the value of the Lagrange variable (λ) according to equation (15); 2). Check the value of the prohibited variable (λ) with the following conditions, If $\lambda > 0$, the optimal model formula uses equation (14) or (13), If $\lambda < 0$, then the optimal model formula uses equation (10), and If $\lambda = 0$, the optimal model formula uses equation (10), equation (14) or (13); 3). Calculate the total cost of inventory using equation (9).

2.6 Numerical experimental procedure model

As the implementation of the proposed EOQ model, we conducted a numerical experiment as a method to validate the model. Numerical experiments were conducted to find out the results of the comparison between the SEOQ 1 model and the proposed SEOQ model 2. The cost component is shown in table 1. This experiment has a variation of 20 different warehouse capacity data. Warehouse capacity is varied with a minimum value of 200 m3 and a maximum value of 2000 m3.

3. Results and discussion

Table 2 shows the results of the SEOQ 1 model and SEOQ 2 model. In SEOQ 1 model, the quantity order is 46 units per order. Furthermore, in the SEOQ 2 model, the number of orders adjusts to the limits of available capacity and room use per unit of product. Based on Table 2, if the available warehouse capacity is relatively small, the order lot size is relatively small. Moreover, the available warehouse capacity is relatively large; the size of the ordering lot is relatively large. The experimental results can be concluded that the smaller the lambda value (λ), the higher the total cost of inventory. Conversely, the higher the value of λ, the greater the total cost of inventory. The optimal total inventory cost (TC) is obtained if the value of $\lambda = 0$. If the boundary value or lambda is > 0, then the optimal model formula is used using equations (14) or (13). The Q value indicates that the order quantity that must be done does not exceed the available warehouse capacity limit. If the lambda value

is <0, then the optimal model formula uses equation (10). It also viewed from the results of the order quantity according to the available warehouse capacity. Based on Table 2, the smaller the warehouse capacity, the higher the total inventory cost. Moreover, if the available warehouse capacity obtains higher, then the total cost of inventory is smaller. The experimental results show that cost emission, waste cost, and warehouse cost are influenced by order quantity (Q). In the cost transportation emission and waste cost, the higher the Q value, the lower the cost transportation emission and waste. It because the frequency of shipping of goods is smaller. Furthermore, the smaller the Q value, the higher the emission transportation costs and waste costs. The results of this experiment are consistent with the research conducted by Battini, et al. [21]. In warehouse costs, the higher the value of Q, the higher the total warehouse costs. Moreover, the smaller the Q value, the smaller the total warehouse cost.

Table 1. Cost Components

Variable	Value	Unit	Variable	Value	Unit
A	100	Rp/order	D	2000	Unit/year
c	350	Rp/unit	β	100	Rp/hour
h	200	Rp/unit/year	v	70	Km/hour
a	50	Rp	γ	20	Rp/unit
b	25	Rp/unit/km	g	40	m^3
d	140	Km	o	12	-
α	0,5	-	γ_0	15	-
			θ	14	-

Table 2. Recapitulates the Results of the SEOQ 1 Model and the SEOQ 2 Model

Model SEOQ 2				Model SEOQ 2			
F	λ	$Q_{(F)}$	$TC_{(h)}$ (Rp)	F	λ	$Q_{(F)}$	$TC_{(h)}$ (Rp)
100	1427,667	8	8432434,8	1100	-36,135	92	8346586,3
200	320,667	17	8363468,2	1200	-38,083	100	8350301,5
300	115,667	25	8343701,5	1300	-39,600	108	8354188,7
400	43,917	33	8336234,8	1400	-40,803	117	8358211,0
500	10,707	42	8333688,2	1500	-41,773	125	8362341,5
600	-7,333	50	8333601,5	1600	-42,568	133	8366559,8
700	-18,211	58	8334920,5	1700	-43,226	142	8370850,5
800	-25,271	67	8337118,2	1800	-43,778	150	8375201,5
900	-30,111	75	8339901,5	1900	-44,245	158	8379603,3
1000	-33,573	83	8343094,8	2000	-44,643	167	8384048,2
				Model SEOQ 1 :		Q=46	8311316

4. Conclusion

This study aims to develop a Sustainable EOQ model. There are two proposed SEOQ models. Model 1 is the SEOQ model with the cost of transportation and warehouse as well as carbon emissions. In the SEOQ 2 model, this model considers the costs of transportation and warehouse, carbon emissions, and capacity. We proposed optimal procedures to solve the problem of Sustainable EOQ with warehouse capacity limits. The results show a useful model to solve inventory problems by considering the costs of transportation and warehouses, carbon emissions, and capacity. The future work is that a sustainable EOQ model can be developed for multi items with several limitations such as warehouse boundaries and capital constraints.

5. References

[1]   G. Kim, K. Wu, and E. Huang, "Optimal inventory control in a multi-period newsvendor problem with non-stationary demand," *Advanced Engineering Informatics,* vol. 29, pp. 139-145, 2015.

[2]   D. M. Utama, "Penentuan Lot Size Pemesanan Bahan Baku Dengan Batasan Kapasitas Gudang," *Jurnal Ilmiah Teknik Industri,* vol. 15, pp. 64-68, 2016.

[3]   D. M. Utama, "Model Penentuan Lot Pemesanan Dengan Mempertimbangkan Unit Diskon dan Batasan Kapasitas Gudang dengan Program Dinamis," *2017,* vol. 18, p. 9, 2017-08-24 2017.

[4]   D. M. Utama, D. P. Wardani, S. T. Halifah, and D. C. Pradikta, "Model Economic Production Quantity dengan Rework Process dan Batasan Gudang," *Jurnal Sistem dan Manajemen Industri,* vol. 3, pp. 43-49, 2019.

[5]   F. Harris, "What quantity to make at once," *The library of factory management,* vol. 5, pp. 47-52, 1915.

[6]   H. M. Wagner and T. M. Whitin, "Dynamic version of the economic lot size model," *Management science,* vol. 5, pp. 89-96, 1958.

[7]   D. M. Utama, "Model Program Dinamis Untuk Lot Size Multi Item Dengan Kendala Kapasitas Gudang," *J@ ti Undip: Jurnal Teknik Industri,* vol. 1, pp. 21-26.

[8]   E. A. Silver, "A heuristic for selecting lot size quantities for the case of a deterministic time-varying demand rate and discrete opportunities for replenishment," *Prod. Inventory Manage.,* vol. 2, pp. 64-74, 1973.

[9]   W. J. Baumol and H. D. Vinod, "An inventory theoretic model of freight transport demand," *Management science,* vol. 16, pp. 413-421, 1970.

[10]   F. P. Buffa and J. I. Reynolds, "The inventory-transport model with sensitivity analysis by indifference curves," *Transportation Journal,* pp. 83-90, 1977.

[11]   M. Bonney and M. Y. Jaber, "Environmentally responsible inventory models: Non-classical models for a non-classical era," *International Journal of Production Economics,* vol. 133, pp. 43-53, 2011.

[12]   J. Qin, X. Bai, and L. Xia, "Sustainable Trade credit and replenishment policies under the cap-and-trade and carbon tax regulations," *Sustainability,* vol. 7, pp. 16340-16361, 2015.

[13]   X. Ma, P. Ji, W. Ho, and C.-H. Yang, "Optimal procurement decision with a carbon tax for the manufacturing industry," *Computers & Operations Research,* vol. 89, pp. 360-368, 2018.

[14]   H. Yang, J. Luo, and H. Wang, "The role of revenue sharing and first-mover advantage in emission abatement with carbon tax and consumer environmental awareness," *International Journal of Production Economics,* vol. 193, pp. 691-702, 2017.

[15]   V. Hovelaque and L. Bironneau, "The carbon-constrained EOQ model with carbon emission dependent demand," *International Journal of Production Economics,* vol. 164, pp. 285-291, 2015.

[16]   N. Absi, S. Dauzère-Pérès, S. Kedad-Sidhoum, B. Penz, and C. Rapine, "The single-item green lot-sizing problem with fixed carbon emissions," *European Journal of Operational Research,* vol. 248, pp. 849-855, 2016.

[17]   S. Tang, W. Wang, H. Yan, and G. Hao, "Low carbon logistics: Reducing shipment frequency to cut carbon emissions," *International Journal of Production Economics,* vol. 164, pp. 339-350, 2015.

[18]   X. Chen, S. Benjaafar, and A. Elomri, "The carbon-constrained EOQ," *Operations Research Letters,* vol. 41, pp. 172-179, 2013.

[19]   M. Y. Jaber, M. Bonney, and H. Jawad, "Comparison between economic order/manufacture quantity and just-in-time models from a thermodynamics point of view," *Computers & Industrial Engineering,* vol. 112, pp. 503-510, 2017.

[20]   A. A. Taleizadeh, V. R. Soleymanfar, and K. Govindan, "Sustainable economic production quantity models for inventory systems with shortage," *Journal of cleaner production,* vol. 174, pp. 1011-1020, 2018.

[21]   D. Battini, A. Persona, and F. Sgarbossa, "A sustainable EOQ model: theoretical formulation and applications," *International Journal of Production Economics,* vol. 149, pp. 145-153, 2014.